

ECO 4354 / 6354 Forecasting

Test 1

Fall 2026 · Tuesday, October 6, 2026

Instructor: Zhan Gao

Name: _____ SMU ID: _____

Instructions

- This is a closed-book-notes exam.
 - Time allowed: 80 minutes.
 - Show your derivation and reasoning. Partial credits are given.
-

Question 1 (50 points)

1.1 (10 points) Consider the linear regression model

$$y_t = \alpha + \beta x_t + u_t, \quad t = 1, 2, \dots, T, \quad (1)$$

where $u_t \sim \text{iid } N(0, 1)$, $x_t \sim \text{iid } N(0, 1)$, and $E(u_t \mid x_1, \dots, x_T) = 0$ for every t .

What is the OLS estimator $\hat{\beta}$ of β ? Write down the formula.

1.2 (10 points) Is $\hat{\beta}$ an unbiased estimator of β ? Explain why or why not.

1.3 (10 points) Now consider the dynamic model

$$y_t = \phi y_{t-1} + \varepsilon_t, \tag{2}$$

where $\varepsilon_t \sim \text{iid } N(0, 1)$ and $|\phi| < 1$. You observe the sample $y_1, y_2, \dots, y_T$ and estimate ϕ by an OLS regression of y_t on y_{t-1} without an intercept.

What is the OLS estimator $\hat{\phi}$ of ϕ ? Is $\hat{\phi}$ unbiased for ϕ ? Explain why or why not.

1.4 (10 points) Suppose you want to use model (2) and the estimator $\hat{\phi}$ from Part 1.3 to construct the optimal forecast of y_{T+3} .

What is the information set Ω_T on which this forecast is based? Write it out explicitly.

1.5 (10 points) Suppose the loss function you care about is

$$L(e) = \frac{1}{2} e^2,$$

where $e = y_{T+3} - \hat{y}_{T+3|T}$ is the forecast error.

What is the optimal forecast of y_{T+3} ? Derive it from model (2).

Question 2 (50 points)**2.1 (10 points)** Let

$$y_t = 0.6 y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \text{iid } N(0, \sigma^2) \text{ with } \sigma^2 = 2,$$

and suppose $\{y_t\}$ is covariance stationary.

Calculate the autocovariance $\gamma(1)$, the autocorrelation $\rho(1)$, and the partial autocorrelations $p(1)$ and $p(2)$.

2.2 (10 points) Let

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t, \quad \varepsilon_t \sim \text{iid } N(0, \sigma^2),$$

and suppose $\{y_t\}$ is covariance stationary. Suppose we already know the autocovariances at the first eleven displacements: $\gamma(\tau) = a_\tau$ for $\tau = 0, 1, 2, \dots, 10$.

Write $\gamma(11)$ as a function of ϕ_1 , ϕ_2 and the a_τ 's. Show your derivation.

2.3 (10 points) Let $\{y_t\}$ be a time series with $E(y_t) = \mu$ for all t , and let

$$\gamma(t, \tau) = \text{Cov}(y_t, y_{t-\tau})$$

denote its autocovariance at date t and displacement τ , for all dates t and all displacements $\tau = 0, 1, 2, \dots$ (so that $\gamma(t, 0) = \text{Var}(y_t)$).

Which of the following candidate autocovariance functions are consistent with covariance stationarity, and which are not? Briefly explain why.

(a) $\gamma(t, \tau) = \frac{1}{2}$

(b) $\gamma(t, \tau) = e^{-\tau/2}$

(c) $\gamma(t, \tau) = \sqrt{1/\tau}$

(d) $\gamma(t, \tau) = t - \tau$ (defined for displacements $0 \leq \tau \leq t$)

2.4 (10 points) Suppose

$$y_t = 1.7 y_{t-1} - 0.72 y_{t-2} + \varepsilon_t, \quad \varepsilon_t \sim WN(0, 2).$$

Is $\{y_t\}$ covariance stationary? Justify your answer.

2.5 (10 points) Figure 1 shows a time series of $T = 500$ observations, together with its sample autocorrelation function, and sample partial autocorrelation function up to displacement 10.

The series was generated by one of the following models:

(A) white noise; (B) AR(1); (C) AR(2); (D) AR(3); (E) AR(4).

Which one? Briefly explain your reasoning.

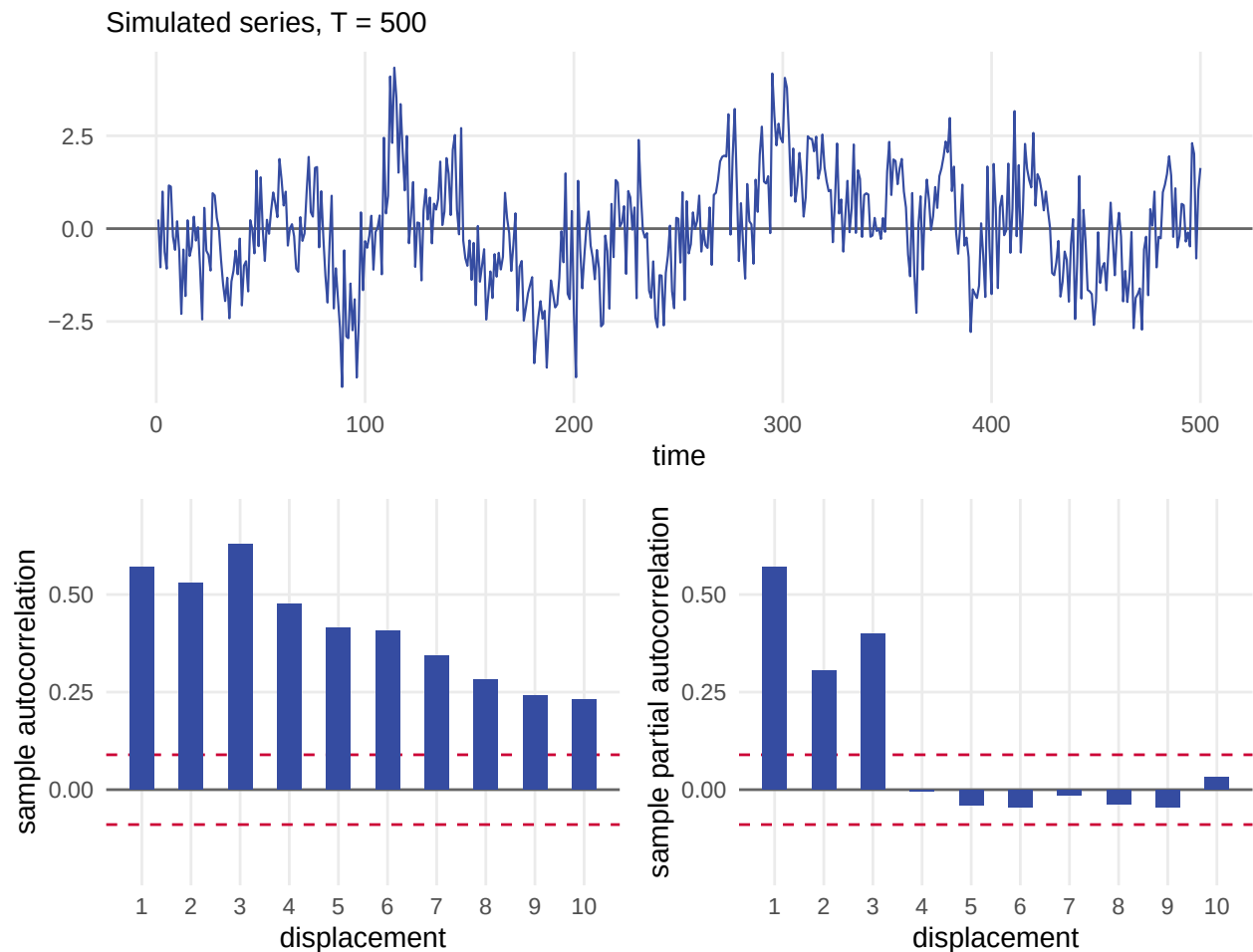


Figure 1: A simulated series (top), its sample autocorrelation function (bottom left) and its sample partial autocorrelation function (bottom right). Dashed lines: plus/minus two standard errors under white noise.

Additional space. If you use this page, indicate clearly which part you are answering.