Eco 7377 Homework

For $i = 1, \cdots, n$, suppose that $(Y_i(0), Y_i(1))$ are (2×1)-random vectors of the potential outcome of state "0" (control) and state "1" (treatment), respectively.

Let T_i denote a binary treatment dummy variable of individual i, where $T_i = 0$ if individual i is controlled (no treatment) and $T_i = 1$ if individual i is treated. Here each individual is either controlled or treated, but not both.

Assumption 1 *We assume that $(Y_i(0), Y_i(1))'$ are iid with mean $(\alpha_0, \alpha_1)'$ and variance $\begin{pmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2 \end{pmatrix}$.*

Assumption 2 *We assume that T_i are iid with mean p, where $p \in [\eta, 1-\eta]$ for some $0 < \eta < 1/2$.*

The observed outcome Y_i is

$$Y_i := Y_i(0)(1 - T_i) + Y_i(1)T_i. \tag{1}$$

For each individual i, the treatment effect is

$$\beta_i = Y_i(1) - Y_i(0).$$

Let

$$\beta_0 := \mathbb{E}(\beta_i) = \alpha_1 - \alpha_0$$

denote the average treatment effect. The goal is to estimate β.

Assumption 3 (Randomized Treatments) *Assumption T_i is independent of $(Y_i(0), Y_i(1))$.*

Please answer to the following questions under Assumptions 1, 2, and 3.

1. To estimate β, suppose that an econometrician sets-up the following simple dummy regressor regression,
$$Y_i = \alpha_0 + \beta_0 T_i + U_1,$$
where $\alpha_0 = \mathbb{E}(Y_i(0))$ and $\beta_0 = \mathbb{E}(\beta_i)$. Write U_i as a function of $(Y_i(0), Y_i(1), T_i)$. [Hint. Manipulate (1).]

2. Is $\mathbb{E}(U_i | T_i = \tau) = 0$ for $\tau = 0, 1$?

3. Find $\mathbb{E}(U_i^2|T_i = \tau)$ for $\tau = 0$ and $\tau = 1$.

   Let $n_0 := \sum_{i=1}^n (1 - T_i)$, the number of individuals whose $T_i = 0$ in the sample. Similarly, let $n_1 := \sum_{i=1}^n T_i$, the number of individuals whose $T_i = 1$ in the sample. Notice that $n = n_0 + n_1$.

4. Show that the OLS estimator, $\widehat{\beta}$, of β_0 in the linear regression equation of question 1 is

$$\widehat{\beta} = \overline{Y}_1 - \overline{Y}_0,$$

   where $\overline{Y}_0 = \frac{\sum_{i=1}^n Y_i(1-T_i)}{n_0}$ and $\overline{Y}_1 = \frac{\sum_{i=1}^n Y_i T_i}{n_1}$

5. Show that $\widehat{\beta}$ is consistent of β_0.

6. Derive the asymptotic distribution of $\widehat{\beta}$.

7. Provide a consistent estimator of the limit variance you derive in question 6.

8. Suppose that $p = 1/2$. Suppose that the econometrician constructs the two-sided 95% confidence interval using the standard error assuming homoskedasticity. What is the asymptotic coverage probabilily of the CI? Is it valid?

   Instead of the random assignment of T_i as in Assumption 3, the treatment is assigned as $T_i = 1$ if $Y_i(1) > Y_i(0)$ and $T_i = 0$ otherwise.

9. Is $\mathbb{E}(U_i|T_i) = 0$?

10. Is $\widehat{\beta}$ is consistent of β_0?